GRAY

Theory of Short Circuits in Alternators

Electrical Engineering

M. S.

1912

THEORY OF SHORT CIRCUITS IN ALTERNATORS

BY

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B. S., University of Illinois, 1911

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

MASTER OF SCIENCE

IN ELECTRICAL ENGINEERING

IN

THE GRADUATE SCHOOL

OF THE

UNIVERSITY OF ILLINOIS

1912

UNIVERSITY OF ILLINOIS

THE GRADUATE SCHOOL

May 31, 1912

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

FRED JAY GRAY

ENTITLED THEORY OF SHORT CIRCUITS IN ALTERNATORS

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF SCIENCE IN ELECTRICAL ENGINEERING.

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THEORY OF SHORT CIRCUITS IN ALTERNATORS.

I. INTRODUCTION.

The theory of the short circuit of an alternator is a good example of the application of mathematics to the study of transient electrical phenomena. It is a splendid illustration of the fact that higher mathematics is of direct use in practical Electrical Engineering and that problems of great complexity can be solved by differential equations.

To be sure, in the solution of this problem, several approximations are made, but the reader must not conclude that the result is only a rough approximation. A careful study into the assumptions will show that the per cent error introduced is small and the error in theory is by necessity much smaller than the error in the determination of the reactance and other constants of the circuit. It is to be regretted that little time was available for tests, therefore the oscillograms, while in many ways excellent, are not always so; for instance, in two cases the record goes off the sheet for a very short distance.

The exciter is assumed to be large enough so that the short circuit does not affect it in any way. This is a fair assumption and simplifies the calculations materially.

In the solution of the numerical problem, use was made of data obtained by Messrs. Harshman, Smith, Sawyer and Weeks in their theses presented in the year nineteen hundred and eleven. The curve for inductance per slot, Plate 2, is merely reproduced from their theses.

II. GENERAL THEORY.

There are two magnetomotive forces in an alternator, one set up by the field and the other set up by the ampere turns in the armature. The latter opposes the field flux more or less completely, depending upon the out of phase current flowing in the armature. At short circuit the armature current is practically ninety (90) degrees out of phase with the electromotive force, so the magnetomotive force of the armature almost entirely counteracts the magnetomotive force of the field, and thus destroys it.

Since, however, there is a reactance in the field, this change of flux cannot take place instantly, but it follows a logarithmic law. The equation for this flux is of the form,

$$\phi = \Phi \, \epsilon^{-\frac{r_0}{x_0}(\theta - \theta_1)}$$

and since the E. M. F. by rotation is proportional to the flux, the voltage follows the same general law, that is,

$$e = E_1 \epsilon^{-\frac{r_0}{x_0}(\theta - \theta_1)} \sin\theta$$

where E_1 corresponds to Φ. θ_1 is the angle of the electromotive force wave at which the short circuit takes place, and r_0 and x_0 are values of the field resistance and reactance which are not as yet determined. The value of the inductance L, and therefore of the reactance, would be easily obtained if there was no iron in the circuit, but the presence of iron complicates matters, for with it the inductance is no longer constant. It varies with the permeability, and also with the position of the coil with respect to the poles. This large variation may be seen from the curve, Plate 2, and so the value used will be only an approximation. This large variation is due partly to mutual induction and, as the change here is rapid, it would not have time to act. If an average is taken it will probably be as close an approximation as can be secured in any such a way.

After the transient period has ceased there is still a current flowing. This may be called the permanent current and the voltage causing it equals $E_2 \sin \theta$. If the maximum value of short circuit current is I, then $E_2 = I\,z$, where z is the true impedance of the armature, but I is not equal to E divided by z. It equals E divided by z_s where z_s is the synchronous impedance. Then $E_2 = E\frac{z}{z_s}$, and since $E_2 + E_1$ must equal E, $E_1 = E(1 - \frac{z}{z_s})$.

Then $E_1 \varepsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)} \sin \theta$ is the transient term of voltage, where "r_0" and "x_0" are respectively the resistance and reactance of the field.

The equation for voltage at any instant is,

$$e = E_1 \varepsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)} \sin\theta + E_2 \sin\theta.$$

If the armature resistance is "r" and its reactance is "x", the armature current may be obtained from the equation,

$$e = ir + L\frac{di}{dt} = ir + x\frac{di}{d\theta} \quad where \quad \theta = \omega t.$$

$$E_1 \varepsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)} \sin\theta + E_2 \sin\theta = ir + x\frac{di}{d\theta}.$$

$$\frac{di}{d\theta} + i\frac{r}{x} = \frac{E_1}{x}\varepsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)} \sin\theta + \frac{E_2}{x}\sin\theta.$$

The solution of this gives

$$i = \varepsilon^{-\frac{r}{x}\theta}\left[\int \varepsilon^{\frac{r}{x}\theta}\left(\frac{E_1}{x}\varepsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\sin\theta\, d\theta + \frac{E_2}{x}\sin\theta\, d\theta\right) + C\right].$$

$$i = \varepsilon^{-\frac{r}{x}\theta}\left[\frac{E_1}{x}\varepsilon^{\frac{r_0}{x_0}\theta_1}\int \varepsilon^{(\frac{r}{x}-\frac{r_0}{x_0})\theta}\sin\theta\, d\theta + \frac{E_2}{x}\int\varepsilon^{\frac{r}{x}\theta}\sin\theta\, d\theta + C\right].$$

Call $\frac{r}{x} - \frac{r_0}{x_0} = \frac{R}{X}$.

$$i = \varepsilon^{-\frac{r}{x}\theta}\left[\frac{E_1}{x}\varepsilon^{\frac{r_0}{x_0}\theta_1}\int\varepsilon^{\frac{R}{X}\theta}\sin\theta\, d\theta + \frac{E_2}{x}\int\varepsilon^{\frac{r}{x}\theta}\sin\theta\, d\theta + C\right].$$

These integrals can be solved by the method commonly known as the "udv" method, that is;

$$\int udv = uv - \int vdu.$$

Then $\int\varepsilon^{\frac{R}{X}\theta}\sin\theta\, d\theta = \frac{X}{Z}\varepsilon^{\frac{R}{X}\theta}\sin(\theta-\beta)$ where $Z = \sqrt{R^2+X^2}$ and $\tan\beta = \frac{X}{R}$.

Also $\int\varepsilon^{\frac{r}{x}\theta}\sin\theta\, d\theta = \frac{x}{z}\varepsilon^{\frac{r}{x}\theta}\sin(\theta-\beta_1)$ where $z = \sqrt{r^2+x^2}$ and $\tan\beta_1 = \frac{x}{r}$.

Therefore,

$$i = \varepsilon^{-\frac{r}{x}\theta}\left[\frac{E_1}{x}\varepsilon^{\frac{r_0}{x_0}\theta_1}\frac{X}{Z}\varepsilon^{\frac{R}{X}\theta}\sin(\theta-\beta) + \frac{E_2}{x}\frac{x}{z}\varepsilon^{\frac{r}{x}\theta}\sin(\theta-\beta_1) + C\right].$$

$$i = \varepsilon^{-\frac{r}{x}\theta}\left[\frac{E_1 X}{x Z}\varepsilon^{\frac{r_0}{x_0}\theta_1}\varepsilon^{(\frac{r}{x}-\frac{r_0}{x_0})\theta}\sin(\theta-\beta) + \frac{E_2}{z}\varepsilon^{\frac{r}{x}\theta}\sin(\theta-\beta_1) + C\right]$$

$$i = \frac{E_1 X}{x Z} \varepsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)} \sin(\theta-\beta) + \frac{E_2}{Z}\sin(\theta-\beta) + C\varepsilon^{-\frac{R}{X}\theta}.$$

The constant may be evaluated from the fact that there is self-induction in the circuit; therefore the current is zero at the moment of short circuit. The switch is closed at an angle θ_1 of the electromotive force wave.

$$0 = \frac{E_1 X}{x Z}\sin(\theta_1-\beta) + \frac{E_2}{Z}\sin(\theta_1-\beta_1) + C\varepsilon^{-\frac{R}{X}\theta_1}.$$

$$C = -\varepsilon^{\frac{R}{X}\theta_1}\left[\frac{E_1 X}{x Z}\sin(\theta_1-\beta) + \frac{E_2}{Z}\sin(\theta_1-\beta_1)\right]$$

$$\text{Then } i = \frac{E_1 X}{x Z}\varepsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\sin(\theta-\beta) + \frac{E_2}{Z}\sin(\theta-\beta_1) - \varepsilon^{-\frac{R}{X}(\theta-\theta_1)}\left[\frac{E_1 X}{x Z}\sin(\theta_1-\beta) + \frac{E_2}{Z}\sin(\theta_1-\beta_1)\right]$$

Substitute for E_1 and E_2 their values in terms of E and let the ratio of true impedance to synchronous impedance be expressed as k. Then $E_1 = E(1-k)$ and $E_2 = Ek$.

$$i = \frac{E}{x}\left[\frac{X}{Z}(1-k)\varepsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\sin(\theta-\beta) + \frac{kx}{z}\sin(\theta-\beta_1) - \frac{X}{Z}(1-k)\varepsilon^{-\frac{R}{X}(\theta-\theta_1)}\sin(\theta_1-\beta) - \frac{kx}{z}\varepsilon^{-\frac{R}{X}(\theta-\theta_1)}\sin(\theta_1-\beta_1)\right]$$

A few approximations can be made to this equation without impairing its accuracy to any large extent. $\frac{R}{X}$ is composed of two terms of which $\frac{r}{x}$ is by far the largest, so we can assume that $\mathrm{Tan}^{-1}\frac{x}{r} = \mathrm{Tan}^{-1}\frac{X}{R}$, that is, $\beta_1 = \beta$. The reactance is very large in comparison to the resistance, so $\frac{x}{z}$ is practically equal to unity. For the same reason $\frac{X}{Z}$ is practically unity. The error introduced in making these assumptions is less than three per cent in most cases.

The equation may now be written,

$$i = \frac{E}{x}\left[(1-k)\varepsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\sin(\theta-\beta) + k\sin(\theta-\beta) - (1-k)\varepsilon^{-\frac{R}{X}(\theta-\theta_1)}\sin(\theta_1-\beta) - k\varepsilon^{-\frac{R}{X}(\theta-\theta_1)}\sin(\theta_1-\beta)\right]$$

The important part of the current curve is at the first instant, so the permanent term k sin $(\theta - \beta)$ may be cancelled with the transient term $k\varepsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\sin(\theta-\beta)$.

Then the final equation for the armature current is,

$$i = \frac{E}{x}\left[\varepsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\sin(\theta-\beta) - \varepsilon^{-\frac{R}{X}(\theta-\theta_1)}\sin(\theta_1-\beta)\right].$$

In a polyphase circuit the current waves for the different branches will be $\frac{2\pi m}{n}$ degrees apart, where "m" is the phase in ques-

tion and "n" the number of phases in the circuit. Then the equation for the armature current of any phase at short circuit is,

$$i = \frac{E}{X}\left[\epsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)} \sin\left(\theta - \beta + \frac{2\pi m}{n}\right) - \epsilon^{-\frac{r}{x}(\theta-\theta_1)} \sin\left(\theta_1 - \beta + \frac{2\pi m}{n}\right)\right].$$

In connection with this equation it is important to remember that the two phase circuit should be calculated as if it were a four phase circuit, because the two phases are a quarter cycle apart.

The instantaneous value of power is equal to the product of the instantaneous current times the voltage at that instant, or P = ei.

But e at any instant equals $E\left[(1-k)\epsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\sin\theta + k\sin\theta\right]$

Since the first cycle is the important one, assume that

$$k\epsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\sin\theta = k\sin\theta.$$

Then, $$e = E\epsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\sin\theta.$$

$$P = \frac{E^2}{X}\sin\theta\left[\epsilon^{-\frac{2r_0}{x_0}(\theta-\theta_1)}\sin(\theta-\beta) - \epsilon^{-\left(\frac{r}{x}+\frac{r_0}{x_0}\right)(\theta-\theta_1)}\sin(\theta_1-\beta)\right]$$

The equation for power of any phase in a polyphase circuit is,

$$P_m = \frac{E^2}{X}\epsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\sin\left(\theta+\frac{2\pi m}{n}\right)\left[\epsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\sin\left(\theta-\beta+\frac{2\pi m}{n}\right) - \epsilon^{-\frac{r}{x}(\theta-\theta_1)}\sin\left(\theta_1-\beta+\frac{2\pi m}{n}\right)\right]$$

$$\sin\left(\theta+\frac{2\pi m}{n}\right)\sin\left(\theta-\beta+\frac{2\pi m}{n}\right) = \frac{1}{2}\left[\cos\beta - \cos\left(2\theta-\beta+\frac{4\pi m}{n}\right)\right].$$

$$\sin\left(\theta+\frac{2\pi m}{n}\right)\sin\left(\theta_1-\beta+\frac{2\pi m}{n}\right) = \frac{1}{2}\left[\cos(\theta-\theta_1+\beta) - \cos\left(\theta+\theta_1-\beta+\frac{4\pi m}{n}\right)\right].$$

The total power of a multiphase circuit equals the summation of the power of the different branches.

But, $\sum_{m=1}^{m=n}\cos\left(2\theta-\beta+\frac{4\pi m}{n}\right) = 0.$ Also $\sum_{m=1}^{m=n}\cos\left(\theta+\theta_1-\beta+\frac{4\pi m}{n}\right) = 0.$

$$P = \frac{E^2}{X}\left[\frac{n}{2}\epsilon^{-\frac{2r_0}{x_0}(\theta-\theta_1)}\cos\beta - \frac{n}{2}\epsilon^{-\left(\frac{r}{x}+\frac{r_0}{x_0}\right)(\theta-\theta_1)}\cos(\theta-\theta_1+\beta)\right].$$

$$P = \frac{E^2 n}{2X}\left[\epsilon^{-\frac{2r_0}{x_0}(\theta-\theta_1)}\cos\beta - \epsilon^{-\left(\frac{r}{x}+\frac{r_0}{x_0}\right)(\theta-\theta_1)}\cos(\theta-\theta_1+\beta)\right].$$

From this equation it may be seen that the power, and therefore the torque on the shaft, at short circuit does not depend upon the angle of closing the switch, but is merely a function of the constants of the circuit and of the time after closing the switch. This is not so with a single phase short circuit as the power depends upon the angle of closing.

These equations have been written for a short circuit of one

coil; for example - from a terminal to the neutral of a three phase "Y" connected machine, but they will hold equally well for a short circuit between two terminals, because the terminal voltage E of a three phase "Y" connected machine is equal to the square root of three times the value per coil. Power in a three phase system equals $\sqrt{3}$ EI, where E and I are the values of line voltage and current respectively. But E equals $\sqrt{3}$ times the value per coil or power P = 3 EI. This is the same as that derived above, for in the equation the factor "n" occurs and it is equal to three for a three phase circuit. This equation is interesting as it shows that the power in a multiphase short circuit is the same whether all the coils or all the lines are short circuited.

In this discussion constant speed has been assumed during the short circuit, so the instantaneous values of torque equal the instantaneous values of power.

Magnetomotive force is equal to current times turns on the armature times the cosine of the space angle. This angle is the same as the E. M. F. angle θ, since it must be 180 degrees from maximum magnetizing to maximum demagnetizing positions of flux and it is also 180 electrical degrees from a north pole to the next south pole of a machine.

Therefore, M. M. F. = I T cos θ

$$M = \frac{TE}{X}\left[\varepsilon^{-\frac{r}{x_o}(\theta-\theta_1)}\cos\theta\,\sin(\theta-\beta) - \varepsilon^{-\frac{r}{x}(\theta-\theta_1)}\cos\theta\,\sin(\theta_1-\beta)\right].$$

This gives the result in ampere-turns. To get this into gilberts multiply by $\frac{4\pi}{10}$.

Flux equals magnetomotive force divided by reluctance.

$$\phi = \frac{4\pi M}{10\,\mathcal{R}}. \qquad Let\ \frac{4\pi\omega}{10^9\,\mathcal{R}} = K.$$

$$Then\ \phi = \frac{KTE\cdot 10^8}{\omega X}\left[\varepsilon^{-\frac{r}{x_o}(\theta-\theta_1)}\cos\theta\,\sin(\theta-\beta) - \varepsilon^{-\frac{r}{x}(\theta-\theta_1)}\cos\theta\,\sin(\theta_1-\beta)\right].$$

The instantaneous values of transient E. M. F. of the field

windings equals $\omega \frac{d\Phi}{d\theta}$.

$$e = \frac{KTE}{X}\left[\frac{d}{d\theta}\left\{\epsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\cos\theta\,\sin(\theta-\beta)\right\} - \sin(\theta_1-\beta)\frac{d}{d\theta}\left(\epsilon^{-\frac{r}{x}(\theta-\theta_1)}\cos\theta\right)\right]$$

$$\frac{d}{d\theta}\left\{\epsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\cos\theta\,\sin(\theta-\beta)\right\} = \epsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\left[\cos(2\theta-\beta) - \frac{r_0}{2x_0}\left\{\sin(2\theta-\beta)-\sin\beta\right\}\right]$$

$$\frac{d}{d\theta}\epsilon^{-\frac{r}{x}(\theta-\theta_1)}\cos\theta = -\frac{z}{x}\epsilon^{-\frac{r}{x}(\theta-\theta_1)}\cos(\theta-\beta).$$

But $\frac{z}{x}$ has been assumed unity and $\beta_1=\beta$.

$$e=\frac{KTE}{X}\left[\epsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\left\{\cos(2\theta-\beta)-\frac{r_0}{2x_0}[\sin(2\theta-\beta)-\sin\beta]\right\}+\epsilon^{-\frac{r}{x}(\theta-\theta_1)}\sin(\theta_1-\beta)\cos(\theta-\beta)\right].$$

$e = i_0 r_0 + x_0\frac{di_0}{d\theta}$ where e, i_0, r_0, and x_0 are values of field voltage, current, resistance and reactance respectively.

$$\text{Then}\quad i_0 = \epsilon^{-\frac{r_0}{x_0}\theta}\left[\int\epsilon^{\frac{r_0}{x_0}\theta}\frac{e}{x_0}\,d\theta + C\right].$$

$$\int \epsilon^{\frac{r_0}{x_0}\theta}\frac{e}{x_0}\,d\theta = \frac{KTE}{Xx_0}\left[\epsilon^{\frac{r_0}{x_0}\theta_1}\left\{\int\cos(2\theta-\beta)d\theta - \frac{r_0}{2x_0}\int\sin(2\theta-\beta)d\theta + \frac{r_0}{2x_0}\sin\beta\int d\theta\right\}\right.$$
$$\left. + \epsilon^{\frac{r}{x}\theta_1}\sin(\theta_1-\beta)\int\epsilon^{-(\frac{r}{x}-\frac{r_0}{x_0})\theta}\cos(\theta-\beta)\,d\theta\right]$$

$$\frac{r}{x}-\frac{r_0}{x_0} = \frac{R}{X}.$$

$$\int\cos(2\theta-\beta)\,d\theta = \frac{1}{2}\sin(2\theta-\beta).$$

$$\int\sin(2\theta-\beta)\,d\theta = -\frac{1}{2}\cos(2\theta-\beta).$$

$$\int\epsilon^{-\frac{R}{X}\theta}\cos(\theta-\beta)\,d\theta = -\frac{X}{Z}\epsilon^{-\frac{R}{X}\theta}\cos\theta = -\epsilon^{-\frac{R}{X}\theta}\cos\theta.\quad \frac{X}{Z}\text{ is assumed unity.}$$

$$\int\epsilon^{\frac{r_0}{x_0}\theta}e\,d\theta = \frac{KTE}{X}\left[\frac{1}{2}\epsilon^{\frac{r_0}{x_0}\theta_1}\left\{\sin(2\theta-\beta) + \frac{r_0}{x_0}[\theta\sin\beta + \cos(2\theta-\beta)]\right\}\right.$$
$$\left. - \epsilon^{\frac{r}{x}\theta_1}\epsilon^{-\frac{R}{X}\theta}\cos\theta\,\sin(\theta_1-\beta)\right]$$

$$\text{But}\ \sin(2\theta-\beta) + \frac{r_0}{x_0}\cos(2\theta-\beta) = \frac{z_0}{x_0}\cos(2\theta-\beta-\alpha)$$

where $\tan\alpha = \frac{x_0}{r_0}$. Assume $\frac{z_0}{x_0}$ equal to unity.

$$i_0 = \frac{KTE}{Xx_0}\left[\frac{1}{2}\epsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\left[\cos(2\theta-\beta-\alpha) + \frac{r_0}{x_0}\theta\sin\beta\right] - \epsilon^{-\frac{r}{x}(\theta-\theta_1)}\sin(\theta_1-\beta)\cos\theta\right] + C\epsilon^{-\frac{r_0}{x_0}\theta}.$$

The constant is found from the fact that there is inductance in the field, so the transient field current is zero at the time the switch is closed, or when $\theta = \theta_1$.

$$0 = \frac{KTE}{Xx_0}\left[\frac{1}{2}\left\{\cos(2\theta_1-\beta-\alpha) + \frac{r_0}{x_0}\theta_1\sin\beta\right\} - \sin(\theta_1-\beta)\cos\theta_1\right] + C\epsilon^{-\frac{r_0}{x_0}\theta_1}.$$

$$C = -\frac{KTE}{Xx_0}\epsilon^{\frac{r_0}{x_0}\theta_1}\left[\frac{1}{2}\left\{\cos(2\theta_1-\beta-\alpha) + \frac{r_0}{x_0}\theta_1\sin\beta\right\} - \sin(\theta_1-\beta)\cos\theta_1\right].$$

$$i_0 = \frac{KTE}{Xx_0}\left[\epsilon^{-\frac{r_0}{x_0}(\theta-\theta_1)}\left\{\frac{1}{2}\left|\cos(2\theta-\beta-\alpha) - \cos(2\theta_1-\beta-\alpha) + \frac{r_0}{x_0}(\theta-\theta_1)\sin\beta\right|\right.\right.$$
$$\left.\left. + \sin(\theta_1-\beta)\cos\theta_1\right\} - \epsilon^{-\frac{r}{x}(\theta-\theta_1)}\sin(\theta_1-\beta)\cos\theta\right].$$

Now since x_0 is very large compared to r_0, α will be very nearly 90 degrees, so assume that this is true. Then,

$$i_0 = \frac{KTE}{XX_0}\Big[\epsilon^{-\frac{r_0}{X_0}(\theta-\theta_1)}\Big\{\frac{1}{2}\Big(\sin(2\theta-\beta)-\sin(2\theta_1-\beta)+\frac{r_0}{X_0}(\theta-\theta_1)\sin\beta\Big)$$
$$+\sin(\theta_1-\beta)\cos\theta_1\Big\} - \epsilon^{-\frac{r}{X}(\theta-\theta_1)}\sin(\theta_1-\beta)\cos\theta\Big].$$

This is the equation for transient field current in a single phase short circuit of an alternator. It is not the entire field current for there is a permanent value that should be added to this to get the total.

This equation must be developed differently for a multiphase short circuit. In general, M. M. F. = I T $\cos(\theta + \frac{2\pi m}{n})$.

Then $M = \frac{ET}{X}\cos(\theta+\frac{2\pi m}{n})\Big[\epsilon^{-\frac{r_0}{X_0}(\theta-\theta_1)}\sin(\theta-\beta+\frac{2\pi m}{n}) - \epsilon^{-\frac{r}{X}(\theta-\theta_1)}\sin(\theta_1-\beta+\frac{2\pi m}{n})\Big].$

The total magnetomotive force is equal to the summation of the separate magnetomotive forces.

$$\text{Total } M = \sum_{m=1}^{m=n} M.$$

$$\cos(\theta+\tfrac{2\pi m}{n})\sin(\theta-\beta+\tfrac{2\pi m}{n}) = \tfrac{1}{2}\Big[\sin(-\beta)+\sin(2\theta-\beta+\tfrac{4\pi m}{n})\Big].$$

$$\cos(\theta+\tfrac{2\pi m}{n})\sin(\theta_1-\beta+\tfrac{2\pi m}{n}) = \tfrac{1}{2}\Big[\sin(\theta_1-\theta-\beta)+\sin(\theta_1+\theta-\beta+\tfrac{4\pi m}{n})\Big].$$

But $\sum_{m=1}^{m=n}\sin(2\theta-\beta+\frac{4\pi m}{n}) = 0.$ Also $\sum_{m=1}^{m=n}\sin(\theta_1+\theta-\beta+\frac{4\pi m}{n}) = 0.$

$\sum_{m=1}^{m=n}\frac{1}{2}\sin(-\beta) = -\frac{n}{2}\sin\beta,$ and $\sum_{m=1}^{m=n}\frac{1}{2}\sin(\theta_1-\theta-\beta) = -\frac{n}{2}\sin(\theta-\theta_1+\beta).$

Therefore, the total magnetomotive force

$$M = \frac{TEn}{2X}\Big[\epsilon^{-\frac{r}{X}(\theta-\theta_1)}\sin(\theta-\theta_1+\beta) - \epsilon^{-\frac{r_0}{X_0}(\theta-\theta_1)}\sin\beta\Big].$$

This gives the magnetomotive force in ampere-turns. To get it into gilberts multiply this by $\frac{4\pi}{10}$.

As before $\phi = \frac{4\pi M}{10\mathcal{R}}$, also let $K = \frac{4\pi\ \omega}{10^9\mathcal{R}}$.

$$\phi = \frac{KTEn\cdot 10^8}{2X\omega}\Big[\epsilon^{-\frac{r}{X}(\theta-\theta_1)}\sin(\theta-\theta_1+\beta) - \epsilon^{-\frac{r_0}{X_0}(\theta-\theta_1)}\sin\beta\Big].$$

$e = \frac{\omega\, d\phi}{10^8 d\theta}$ equals the voltage of the field windings.

$$e = \frac{KTEn}{2X}\Big[\frac{d}{d\theta}\Big\{\epsilon^{-\frac{r}{X}(\theta-\theta_1)}\sin(\theta-\theta_1+\beta)\Big\} - \sin\beta\frac{d}{d\theta}\epsilon^{-\frac{r_0}{X_0}(\theta-\theta_1)}\Big]$$

$$\frac{d}{d\theta}\Big\{\epsilon^{-\frac{r}{X}(\theta-\theta_1)}\sin(\theta-\theta_1+\beta)\Big\} = \epsilon^{-\frac{r}{X}(\theta-\theta_1)}\Big[\cos(\theta-\theta_1+\beta)-\frac{r}{X}\sin(\theta-\theta_1+\beta)\Big].$$

$$e = \frac{KTEn}{2X}\Big[\epsilon^{-\frac{r}{X}(\theta-\theta_1)}\Big\{\cos(\theta-\theta_1+\beta)-\frac{r}{X}\sin(\theta-\theta_1+\beta)\Big\}$$
$$+\frac{r_0}{X_0}\epsilon^{-\frac{r_0}{X_0}(\theta-\theta_1)}\sin\beta\Big].$$

But $\cos(\theta-\theta_1+\beta) - \frac{r}{X}\sin(\theta-\theta_1+\beta) = -\frac{z}{X}\sin(\theta-\theta_1)$ if β_1 is assumed equal to β.

$$\text{Then } e = \frac{KTEn}{2X}\left[\frac{r_0}{X_0}\varepsilon^{-\frac{r_0}{X_0}(\theta-\theta_1)}\sin\beta - \frac{z}{X}\varepsilon^{-\frac{r}{X}(\theta-\theta_1)}\sin(\theta-\theta_1)\right]$$

$$e = i_0 r_0 + X_0\frac{di_0}{d\theta} \quad \text{or} \quad \frac{di_0}{d\theta} + i_0\frac{r_0}{X_0} = \frac{e}{X_0}$$

$$i_0 = \varepsilon^{-\frac{r_0}{X_0}\theta}\left[\int \varepsilon^{\frac{r_0}{X_0}\theta}\frac{e}{X_0}\,d\theta + C\right].$$

$$\int \varepsilon^{\frac{r_0}{X_0}\theta}\frac{e}{X_0}\,d\theta = \frac{KTEn}{2XX_0}\left[\frac{r_0}{X_0}\sin\beta\int\varepsilon^{\frac{r_0}{X_0}\theta_1}d\theta - \frac{z}{X}\varepsilon^{\frac{r}{X}\theta_1}\int\varepsilon^{-\left(\frac{r}{X}-\frac{r_0}{X_0}\right)\theta}\sin(\theta-\theta_1)\,d\theta\right].$$

$$\frac{r}{X} - \frac{r_0}{X_0} = \frac{R}{\mathcal{X}}.$$

$$\int\varepsilon^{-\frac{R}{\mathcal{X}}\theta}\sin(\theta-\theta_1)\,d\theta = -\frac{\mathcal{X}}{Z}\,\varepsilon^{-\frac{R}{\mathcal{X}}\theta}\sin(\theta-\theta_1+\beta).$$

$$\int\varepsilon^{\frac{r_0}{X_0}\theta}\frac{e}{X_0}\,d\theta = \frac{KTEn}{2XX_0}\left[\frac{r_0}{X_0}\theta\,\varepsilon^{\frac{r_0}{X_0}}\sin\beta + \frac{\mathcal{X}}{Z}\frac{z}{X}\varepsilon^{\frac{r}{X}\theta_1}\varepsilon^{-\frac{R}{\mathcal{X}}\theta}\sin(\theta-\theta_1+\beta)\right]$$

$$\text{Assume } \frac{\mathcal{X}}{Z}\frac{z}{X} = \text{unity}.$$

$$i_0 = \frac{KTEn}{2XX_0}\left[\frac{r_0\theta}{X_0}\varepsilon^{-\frac{r_0}{X_0}(\theta-\theta_1)}\sin\beta + \varepsilon^{-\frac{r}{X}(\theta-\theta_1)}\sin(\theta-\theta_1+\beta)\right] + C\varepsilon^{-\frac{r_0}{X_0}\theta}.$$

When the switch is closed - that is, when $\theta = \theta_1$ - the transient field current equals zero.

$$0 = \frac{KTEn}{2XX_0}\left[\frac{r_0\theta_1}{X_0}\sin\beta + \sin\beta\right] + C\varepsilon^{-\frac{r_0}{X_0}\theta_1}$$

$$C = -\frac{KTEn}{2XX_0}\varepsilon^{\frac{r_0}{X_0}\theta_1}\sin\beta\left(\frac{r_0\theta_1}{X_0}+1\right).$$

$$\text{Then } i_0 = \frac{KTEn}{2XX_0}\left[\frac{r_0\theta}{X_0}\varepsilon^{-\frac{r_0}{X_0}(\theta-\theta_1)}\sin\beta + \varepsilon^{-\frac{r}{X}(\theta-\theta_1)}\sin(\theta-\theta_1+\beta) - \varepsilon^{-\frac{r_0}{X_0}(\theta-\theta_1)}\left(1+\frac{r_0\theta_1}{X_0}\right)\sin\beta\right]$$

$$i_0 = \frac{KTEn}{2XX_0}\left[\varepsilon^{-\frac{r_0}{X_0}(\theta-\theta_1)}\sin\beta\left\{\frac{r_0}{X_0}(\theta-\theta_1)-1\right\} + \varepsilon^{-\frac{r}{X}(\theta-\theta_1)}\sin(\theta-\theta_1+\beta)\right]$$

This gives the instantaneous values of field current set up by the short circuit and is not the entire field current. There is a permanent field current flowing that should be added to this to find the total field current. If n is taken as four for a two phase circuit, then E must be half the value as read because that is the only way in which four phases can be obtained. Then each phase is considered as divided into two parts, thus making two phases 180 degrees apart.

III. DESCRIPTION OF MACHINE AND TESTS.

For obtaining data to check these equations, machine number 95,584 Electrical Engineering Laboratory, University of Illinois, was used. It was manufactured by the General Electric Company and is rated as three phase, six pole revolving field, 7.5 K. W., 20 amperes, 220 volts, and 1200 R. P. M. The armature winding is divided into six parts and the terminals of each part are brought out to a connection block so they may be connected any way desired. This enables the machine to be used equally well single phase, two, three or six phase. A picture of the machine as well as a vector diagram of the coils connected for two phase service is shown in Plate 1.

Tests of the machine conducted in the past have shown the machine to have a straight line saturation curve for above normal voltage per coil. It is capable of carrying larger currents than its rated capacity per coil, and thus may be run at a large overload power output.

In running the tests, 220 volts was used with three coils in series, although it is rated 220 volts, three phase.

In this machine the value of reactance for three windings in series was found to be about 2.5 times as great when the winding is under the pole as when it is between the poles, due to its peculiar construction. The ratio will vary with the design of the machine, depending upon the distribution of the winding, the arc of the pole shoe, the length of the poles and the length of the air gap.

The field winding consists of 385 series turns of wire per coil, or 2310 total series turns.

To obtain pictures of the instantaneous value of the armature and field current waves at short circuit, two General Electric Com-

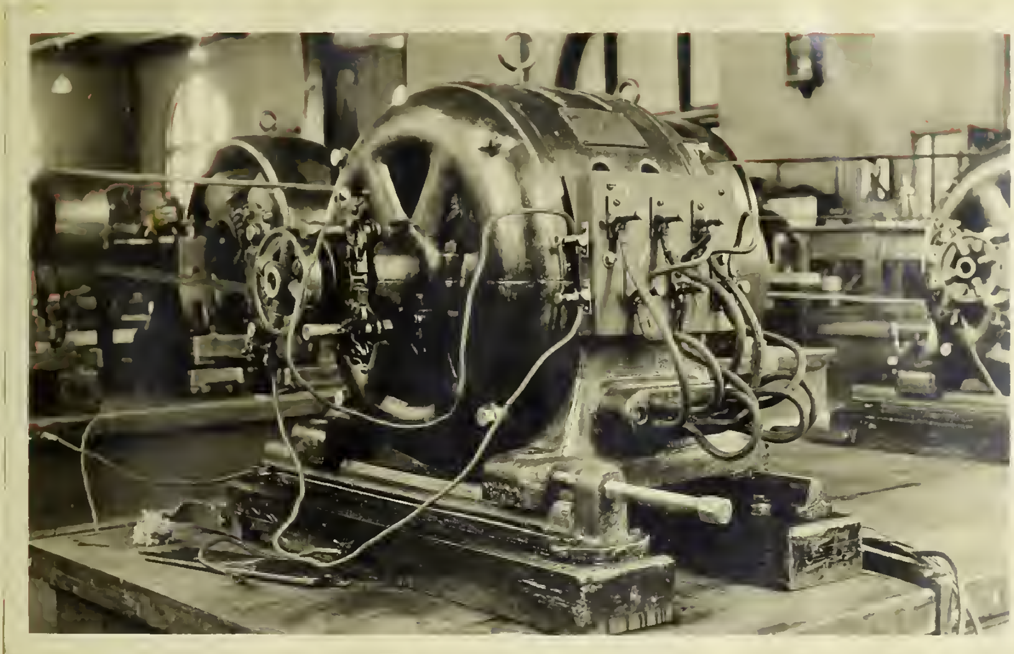
pany oscillographs were used. For the two phase short circuit five waves were wanted so three were taken on one oscillograph and two on the other. The five waves were voltage and current of both phases and field current. The two shutters were opened at the same time since they were in series, so when the contact maker opened one it also opened the other. In the same circuit with the shutters was a relay so arranged that it pulled out a prop and allowed a switch to close when the shutter opened. This switch short circuited the two phases. The voltage waves showed at what angle of the E. M. F. wave the short circuit occurred.

When a single phase short circuit was obtained, only one oscillograph was used and three waves were obtained on it. The relay was connected in series with the shutter as before.

Calibration lines were put on all films taken so that the actual values of current may be calculated at any instant.

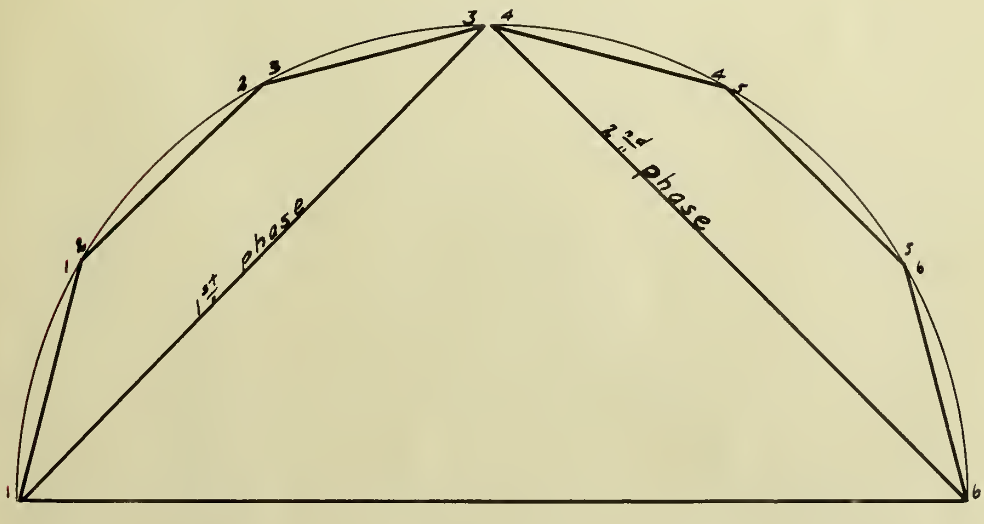
PLATE 1.

Picture of Machine.



Vector Diagram of the Six Windings Connected

as in the Test Made.



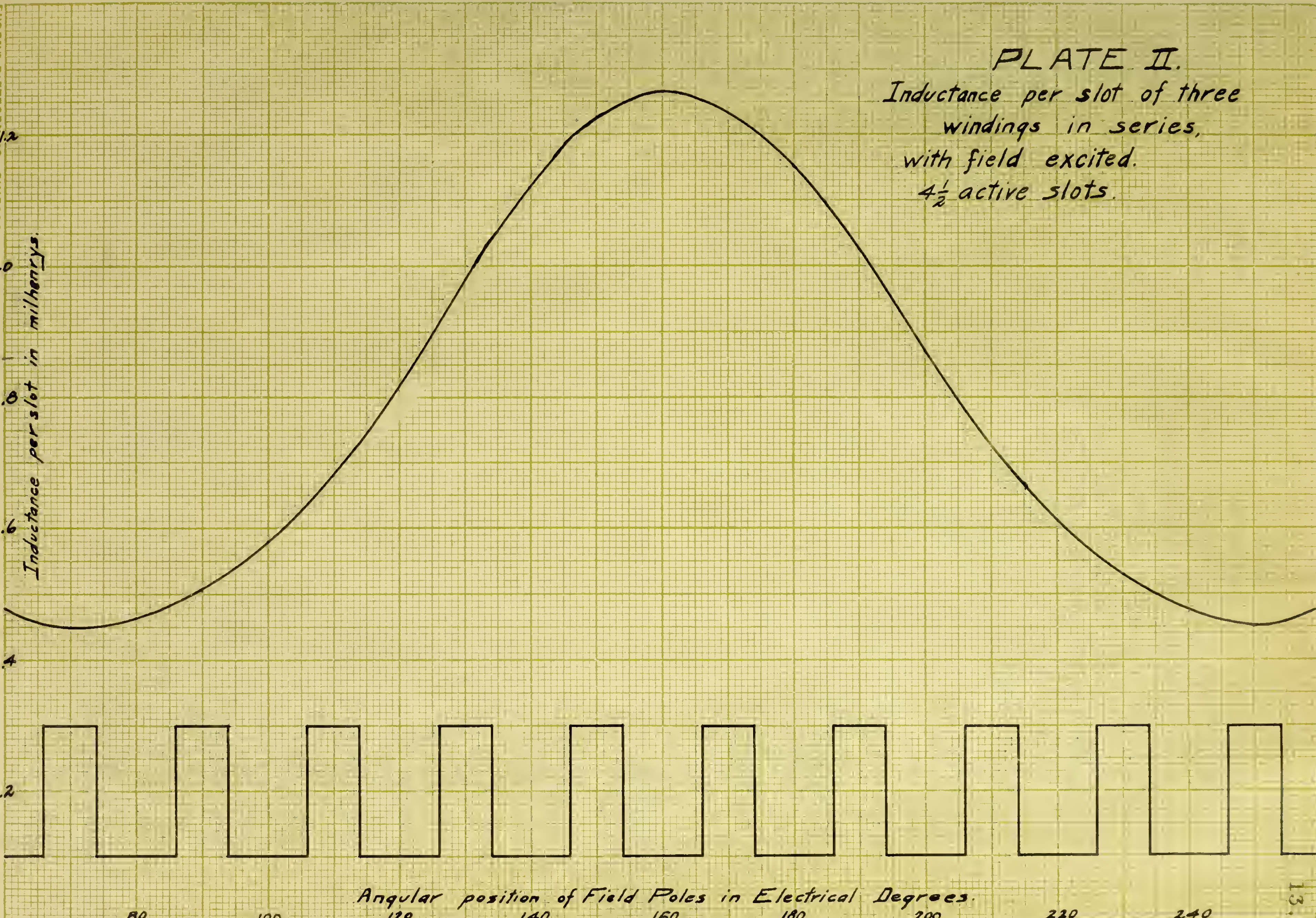
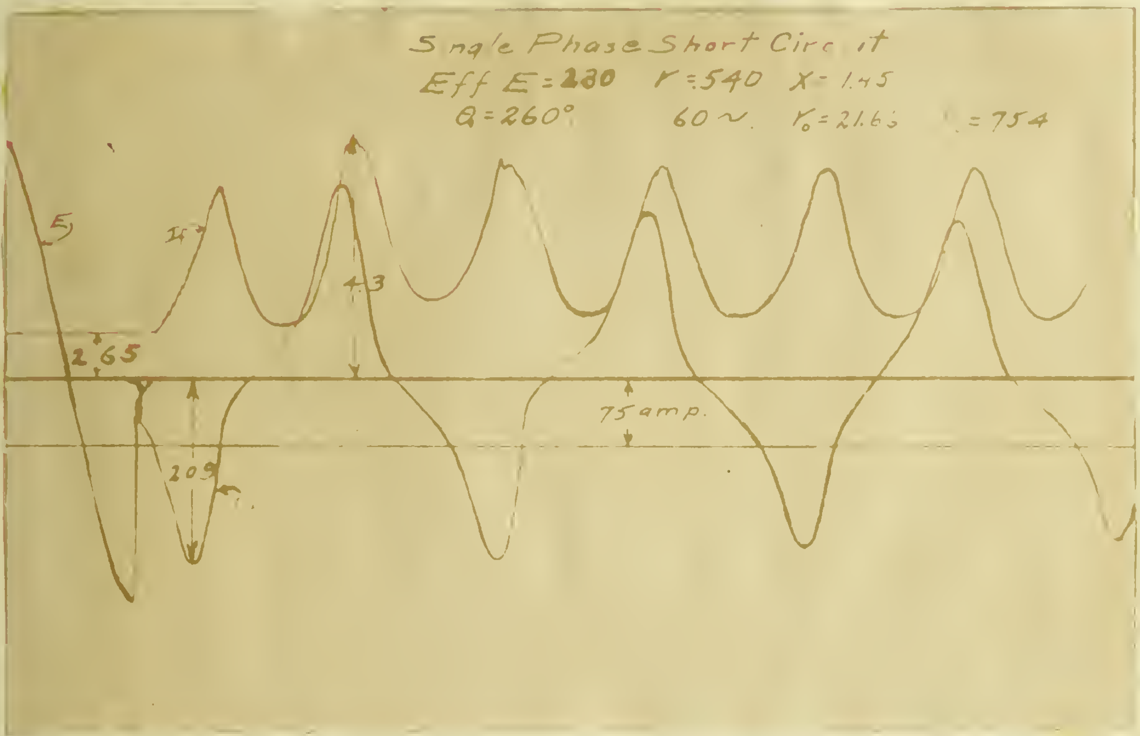


PLATE II.
Inductance per slot of three windings in series, with field excited. $4\frac{1}{2}$ active slots.

PLATE 3.

Switch Closed at Angle of E. M. F. Wave for Least Currents.



Switch Closed at Angle of E. M. F. Wave for Large Currents.

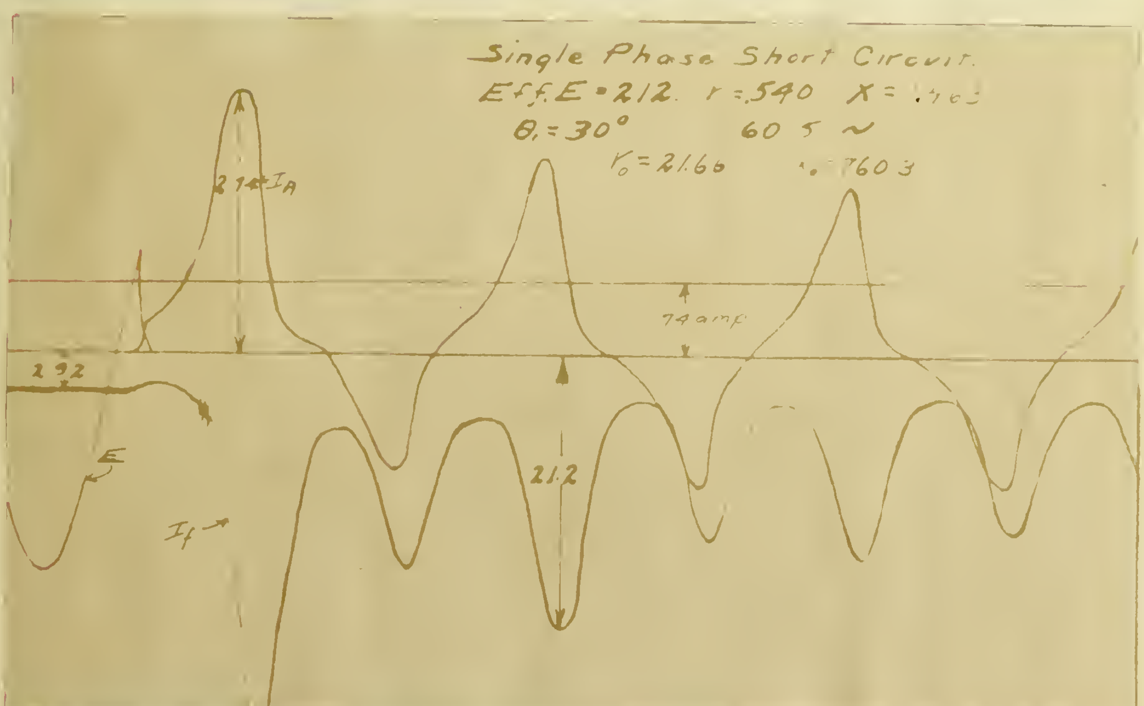
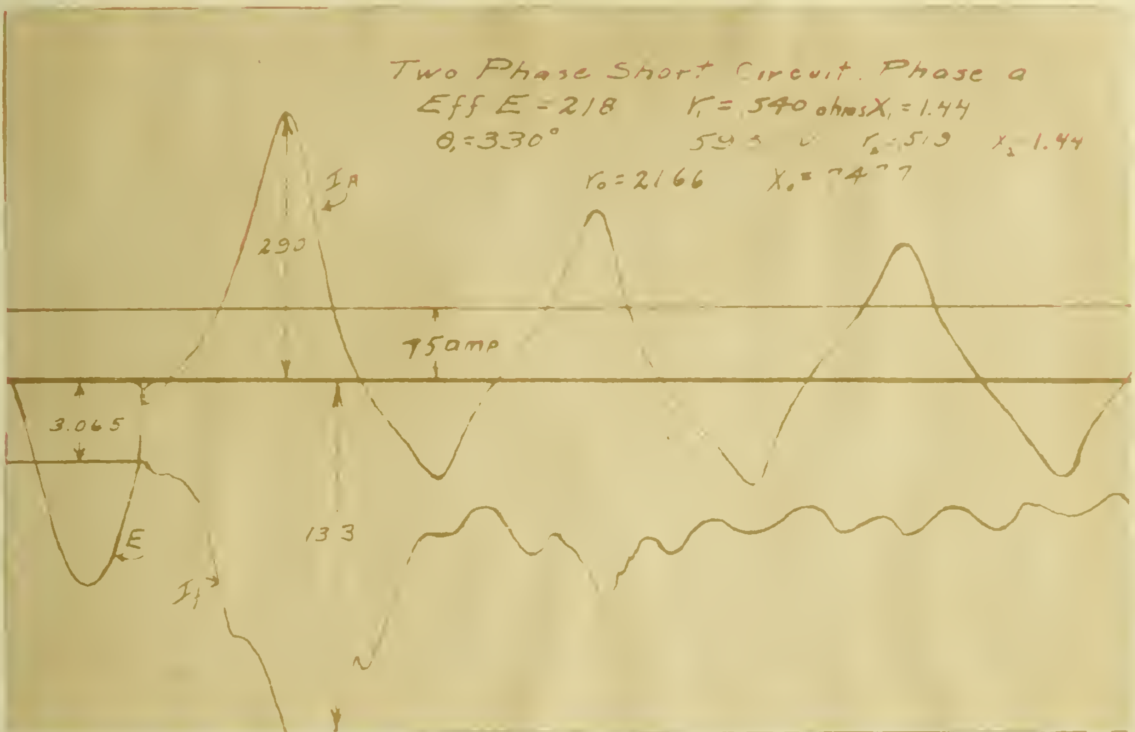


PLATE 4.


Two Phase Short Circuit. Phase a
Eff E = 218
r_1 = .540 ohms X_1 = 1.44
θ = 330°
r_2 = .519
X_2 1.44
r_0 = 2166
I_A
290
75 amp
3.065
E
133
I_f

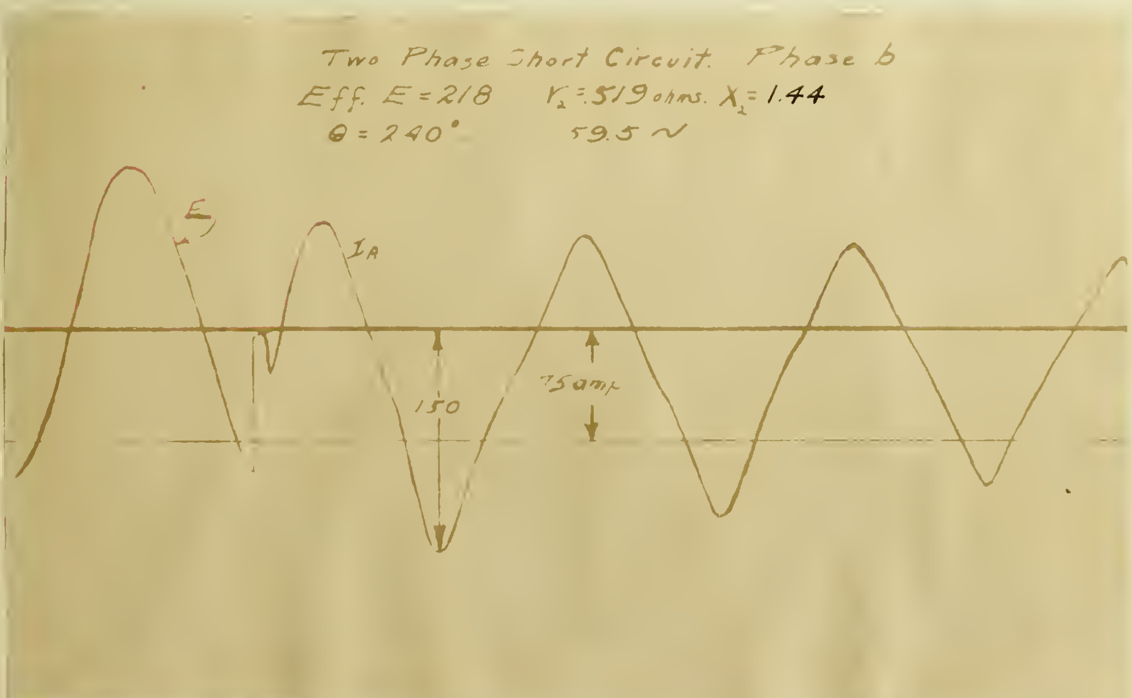
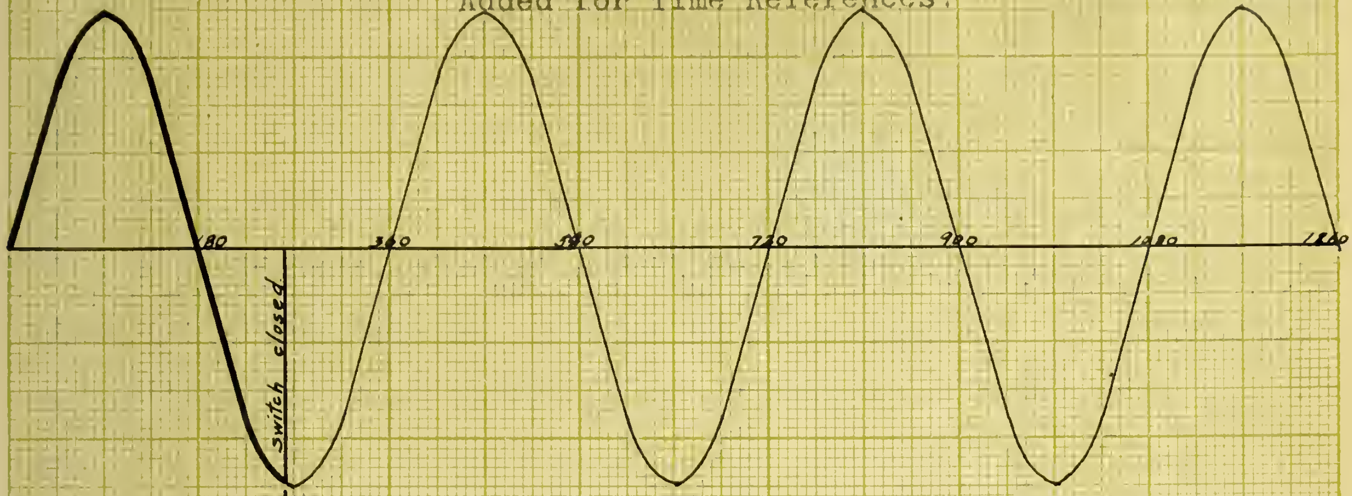
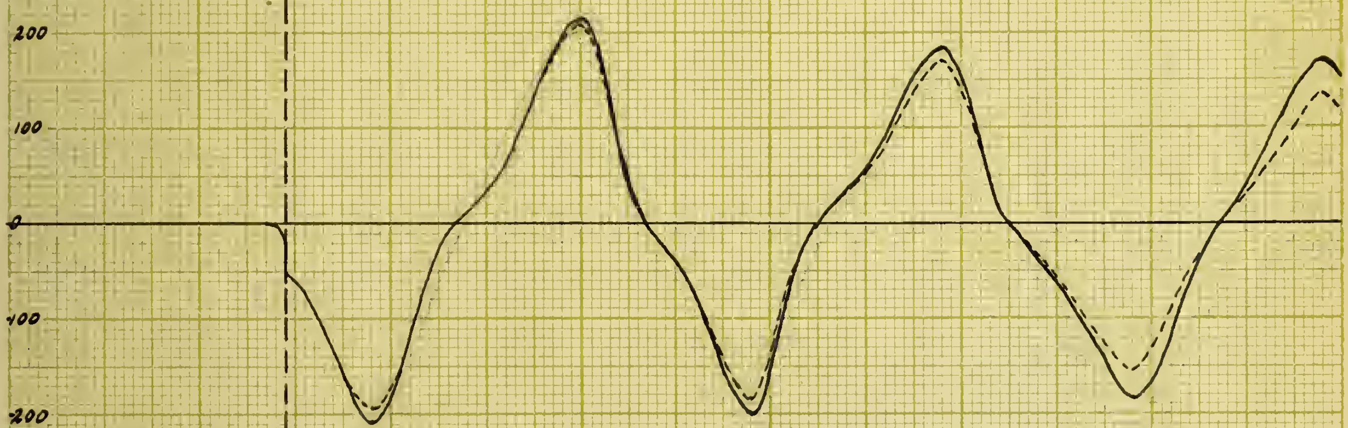

Two Phase Short Circuit. Phase b
Eff. E = 218
r_2 = .519 ohms. X_2 = 1.44
θ = 240°
59.5 ~
E
I_A
150
75 amp

PLATE 5.

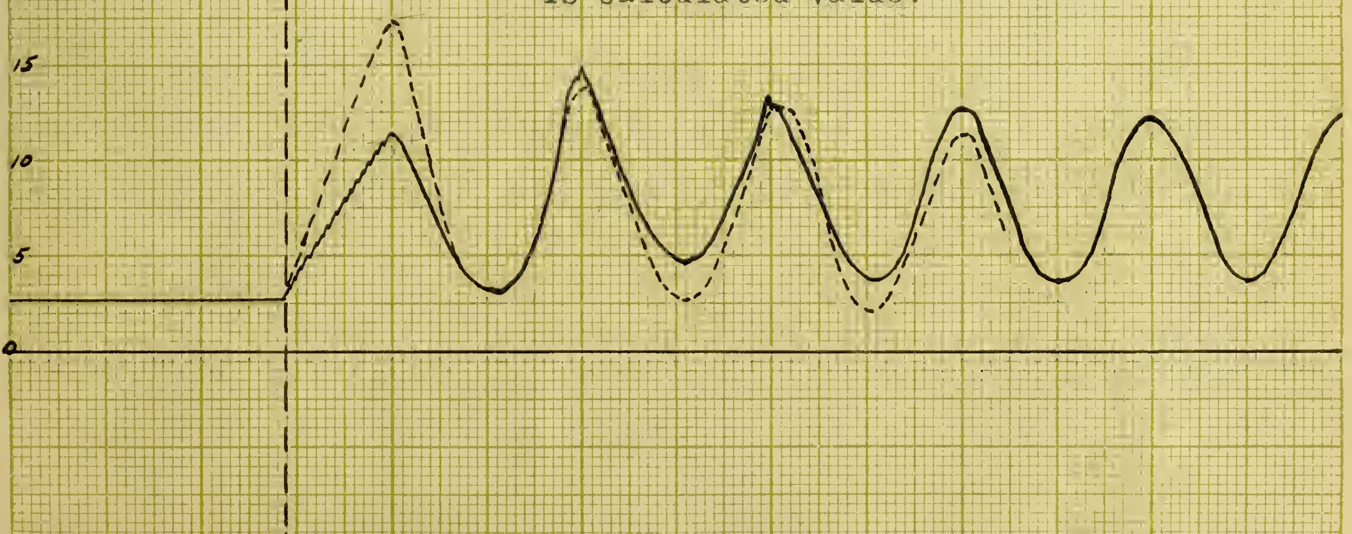
E.M.F. Wave - Dark Part as Shown on Oscillogram; - Light Part Added for Time References.



Armature Current - Full Line is Value as Taken from Oscillograph Record; - Dotted Line is Calculated Value.



Field Current - Full Line is Experimental Value; - Dotted Line is Calculated Value.



V. CONCLUSION.

As can be seen from Plate 5, the calculated results checked fairly closely with the results obtained from the test. The first peak of the field current curve is quite different from the calculated value, but in all the other cases the difference is small. It must be considered in judging this method that the equation is only meant to be approximate. There are many uncertain quantities in this phenomenon, such as the value of armature reactance and the angle of the E. M. F. wave at which the short circuit occurred. For transient phenomena of this kind an approximate formula is all that is necessary, since if the information is wanted for the purpose of guarding against injury, a difference of five per cent would not be very objectionable.

As has been stated in the theory, the power or torque on the shaft of the polyphase short circuit is not affected by the angle of the E. M. F. wave at which short circuit occurs. The same is true of the induced E. M. F. in the field windings and therefore of the field current. This is very interesting and is hardly what a person would expect from a casual survey of the subject. In a polyphase short circuit the value of the armature current in any phase depends upon the angle of the E. M. F. wave at short circuit. In a single phase short circuit the value of armature current, field current, power, torque and induced E. M. F. in the field windings all depend upon the angle of the E. M. F. wave at short circuit.